INFO I201
Midterm II

Tuesday, June 11, 2013
Duration: 80 minutes.

Instructions
1. Answer each question on a new page in your blue books.
2. You are not allowed to use any papers or notes except for one letter size sheet you are allowed to bring to exam.
3. Make sure you write LEGIBLY and give enough explanation whenever it is due.

Q1. (6 pts) Let $A = \{\{0\}, 1, \{0,1\}\}$ and $B = \{0, \{1\}\}$.

(a) Find $A \cap B, B - A, A \cup B$, and $A - B$.

(b) Find $\mathcal{P}(B)$ (the power set of B).

(c) List two elements in $\mathcal{P}(A)$.

(d) Find $\mathcal{P}(A) \cap B$.

(e) List two elements in the set $A \times B$.

Q2. (2 pts) Let $\mathcal{L}$ be a first order language with two binary predicate symbols P and Q, and one ternary predicate symbol T. Consider the formulas

- $\phi_1 \equiv \forall x(\exists y \forall z T(x,y,z) \wedge \exists z \forall y T(x,y,z))$
- $\phi_2 \equiv \forall x \exists y(P(x,y)) \longrightarrow \forall x Q(x,y)$

Determine the free and bound occurrences of all the variables in formulas ϕ_1 and ϕ_2.

Q3. (4 pts) Let $\mathcal{L}$ be a first order language with three unary predicate symbols A, B, and C. Consider the formulas below.

$\phi_1 \equiv \exists x\, A(x)$
$\phi_2 \equiv \exists x\, B(x)$
$\phi_3 \equiv \exists x\, C(x)$
$\phi_4 \equiv \forall x\, (A(x) \longrightarrow B(x))$
$\phi_5 \equiv \forall x\, (B(x) \longrightarrow C(x))$

Design a model $M = (\mathbb{Z}, I)$ such that all formulas $\phi_1, \phi_2, \phi_3, \phi_4$, and ϕ_5 are true.

Q4. (4 pts) (i) Let A, B and C be sets. Show that $(A - B) - C \subseteq A \cap \overline{B}$.

(ii) Let A and B be sets. Prove or disprove: $A \cap B \neq \emptyset$ implies that $A \subseteq \overline{B}$.

(iii) Let A, B and C be sets. Prove or disprove: $A - B \neq B \cap C$ implies that $A \neq B$.

Q5. (5 pts) Consider a first order language $\mathcal{L}$ that consists of one binary predicate symbol P. Also consider the following formulas in this language:

- $\phi_1 \equiv \exists y \forall x P(x, y) \longrightarrow \forall x \exists y P(x, y)$
- $\phi_2 \equiv \forall x \exists y P(x, y) \longrightarrow \exists y \forall x P(x, y)$

Find one **single** model $M = (U, I)$ with $U = \{a, b, c, d\}$ that makes ϕ_1 true and ϕ_2 false.

Q6. (4 pts) Consider a first order language $\mathcal{L}$ that consists of one unary predicate symbol Q. Let ϕ be the formula:

$$\forall x \forall y[(Q(x) \wedge Q(y)) \longrightarrow (x = y)]$$

(i) Let $U = \{a, b, c, d, e\}$ and $I(Q) = \{\}$. Is ϕ valid in this model?

(ii) Let $U' = \{a, b, c, d, e\}$ and $I'(Q) = \{d\}$. Is ϕ valid in this model?